Scaling Data Sample annual salaries (in thousands of dollars) for employees at a company are listed.

42 36 48 51 39 39 42 36 48 33 39 42 45

(a) Find the sample mean and sample standard deviation.

(b) Each employee in the sample is given a 5% raise. Find the sample mean and sample standard deviation for the revised data set.

(c) To calculate the monthly salary, divide each original salary by 12. Find the sample mean and sample standard deviation for the revised data set.

(d) What can you conclude from the results of (a), (b), and (c)?

Note: Students can either use a calculator or do it manually. Calculator is allowed. Manual solution is as follows:

Given:

$$42, 36, 48, 51, 39, 39, 42, 36, 48, 33, 39, 42, 45$$

(a) The mean is the sum of all values divided by the number of values:

$$\overline{x} = \frac{\sum x}{n} = \frac{42+36+48+51+39+39+42+36+48+33+39+42+45}{13} = \frac{540}{13} \approx 41.5385$$

n is the number of values in the data set.

$$n = 13$$

The variance is the sum of squared deviations from the mean divided by $n-1$:

$$s^2 = \frac{\sum (x-\overline{x})^2}{n-1}$$

$$= \frac{(42-41.5385)^2 + (36-41.5385)^2 + (48-41.5385)^2 + (51-41.5385)^2 + (39-41.5385)^2}{+(39-41.5385)^2 + (42-41.5385)^2 + (36-41.5385)^2 + (48-41.5385)^2}$$

$$+ \frac{+(33-41.5385)^2 + (39-41.5385)^2 + (42-41.5385)^2 + (45-41.5385)^2}{13-1}$$

$$\approx 28.2692$$

The standard deviation is the square root of the variance:

$$s = \sqrt{28.2692} \approx 5.3169$$

(b) Every individual receives a 5% raise.

$$
\begin{aligned}
42 + 5\% \cdot 42 &= 42 + 0.05 \cdot 42 = 44.1 \\
36 + 5\% \cdot 36 &= 36 + 0.05 \cdot 36 = 37.8 \\
48 + 5\% \cdot 48 &= 48 + 0.05 \cdot 48 = 50.4 \\
51 + 5\% \cdot 51 &= 51 + 0.05 \cdot 51 = 53.55 \\
39 + 5\% \cdot 39 &= 39 + 0.05 \cdot 39 = 40.95 \\
39 + 5\% \cdot 39 &= 39 + 0.05 \cdot 39 = 40.95 \\
42 + 5\% \cdot 42 &= 42 + 0.05 \cdot 42 = 44.1 \\
36 + 5\% \cdot 36 &= 36 + 0.05 \cdot 36 = 37.8 \\
48 + 5\% \cdot 48 &= 48 + 0.05 \cdot 48 = 50.4 \\
33 + 5\% \cdot 33 &= 33 + 0.05 \cdot 33 = 34.65 \\
39 + 5\% \cdot 39 &= 39 + 0.05 \cdot 39 = 40.95 \\
42 + 5\% \cdot 42 &= 42 + 0.05 \cdot 42 = 44.1 \\
45 + 5\% \cdot 45 &= 45 + 0.05 \cdot 45 = 47.25
\end{aligned}
$$

The mean is the sum of all values divided by the number of values:

$$
\begin{aligned}
\overline{x} &= \frac{\sum x}{n} \\
&= \frac{44.1 + 37.8 + 50.4 + 53.55 + 40.95 + 40.95 + 44.1 + 37.8 + 50.4 + 34.65 + 40.95 + 44.1 + 47.25}{13} \\
&= \frac{567}{13} \\
&\approx 43.6154
\end{aligned}
$$

n is the number of values in the data set.

$$n = 13$$

The variance is the sum of squared deviations from the mean divided by $n - 1$:

$$
\begin{aligned}
s^2 &= \frac{\sum (x - \overline{x})^2}{n - 1} \\
&= \frac{(44.1 - 43.6154)^2 + (37.8 - 43.6154)^2 + (50.4 - 43.6154)^2 + (53.55 - 43.6154)^2 + (40.95 - 43.6154)^2}{+(40.95 - 43.6154)^2 + (44.1 - 43.6154)^2 + (37.8 - 43.6154)^2 + (50.4 - 43.6154)^2} \\
&+ \frac{+(34.65 - 43.6154)^2 + (40.95 - 43.6154)^2 + (44.1 - 43.6154)^2 + (47.25 - 43.6154)^2}{13 - 1} \\
&\approx 31.1668
\end{aligned}
$$

The standard deviation is the square root of the variance:

$$s = \sqrt{31.1668} \approx 5.5827$$

(c) Every salary is divided by 12 (to determine the monthly salary instead of the yearly salary)

$$
\begin{aligned}
42/12 &= 3.5 \\
36/12 &= 3 \\
48/12 &= 4 \\
51/12 &= 4.25 \\
39/12 &= 3.25 \\
39/12 &= 3.25 \\
42/12 &= 3.5 \\
36/12 &= 3 \\
48/12 &= 4 \\
33/12 &= 2.75 \\
39/12 &= 3.25 \\
42/12 &= 3.5 \\
45/12 &= 3.75
\end{aligned}
$$

The mean is the sum of all values divided by the number of values:

$$\overline{x} = \frac{\sum x}{n} = \frac{3.5+3+4+4.25+3.25+3.25+3.5+3+4+2.75+3.25+3.5+3.75}{13} = \frac{45}{13} \approx 3.4615$$

n is the number of values in the data set.

$$n = 13$$

The variance is the sum of squared deviations from the mean divided by $n-1$:

$$
\begin{aligned}
s^2 &= \frac{\sum (x-\overline{x})^2}{n-1} \\
&= \frac{(3.5-3.4615)^2+(3-3.4615)^2+(4-3.4615)^2+(4.25-3.4615)^2+(3.25-3.4615)^2}{} \\
&\frac{+(3.25-3.4615)^2+(3.5-3.4615)^2+(3-3.4615)^2+(4-3.4615)^2}{} \\
&+ \frac{+(2.75-3.4615)^2+(3.25-3.4615)^2+(3.5-3.4615)^2+(3.75-3.4615)^2}{13-1} \\
&\approx 0.1963
\end{aligned}
$$

The standard deviation is the square root of the variance:

$$s = \sqrt{0.1963} \approx 0.4431$$

(d) We note:

$$41.5385 + 5\% \cdot 41.5385 = 41.5385 + 0.05 \cdot 41.5385 = 43.6154$$
$$5.3169 + 5\% \cdot 5.3169 = 5.3169 + 0.05 \cdot 5.3169 = 5.5827$$

Thus we note that if all data values increase by 5%, then the mean and standard deviation also increase by 5%.

In general, if all data values are multiplied by the same constant, then the mean and standard deviation are also multiplied by this constant.

We also note:

$$41.5385/12 = 3.4615$$
$$5.3169/12 = 0.4431$$

Thus we note that if all data values are divided by 12, then the mean and standard deviation is also divided by 12.

In general, if all data values are divided by the same constant, then the mean and standard deviation are also divided by this constant.

5 points
Part A: Students get 1point for finding sample mean and sample standard deviation
Part B: Students get 1 point for finding sample mean and standard deviation after 5% increase
Part C: Students get 1 point for finding mean and standard deviation on a monthly base
Part D: Students get 1 point for explaining that mean and standard deviation increases by 5% compare to the mean and standard deviation in Part A.
Students get 1 point for explaining that mean and standard deviation decreases by 1/12 since the it's being divided by the same constant

Genesis Convert Table

Task Point	Genesis Score
0	55
1	59
2	69
3	79
4	89
5	100